ABSTRACT.

The objective in this document is to establish a set of scientific competences in order to get the frontiers of research in magnetohydrodynamics mainly orientated to nuclear fusion.

FOUNDATIONS.

1. History.

According to Teller, E [82]: “It seems hard to believe that only half a century has passed since Atkinson and Houtermans (1929) proposed that the energy of the sun is released by thermonuclear reactions. Shortly after this important publication, a young and not yet renowned physicist, George Gamow, reported this suggestion at a meeting in Leningrad. After the meeting, Bukharin, a leading member of the Communist party, came to Gamow with a proposal. The whole electrical output of Leningrad could be made available to Gamow for one hour each night if he would undertake to reproduce on earth what was happening in the sun. Gamow, a physicist of unusual taste and common sense, did not accept the offer.” Extensive details of the history of nuclear fusion are in [20, 66, 89].

2. An introduction to general paths of reasoning in MHD.

Plasma physic has a considerable complexity, we give in this section a series of different paths of reasoning beginning with the most readable material, some of them of advanced nature.

A bridge between mathematicians and physicians is given by Artsimovich, L. A. in [7] where this are the main topics:

• Definition of plasma. • Motions of electrons and ions in plasmas in the absence of external fields. • Plasma behavior in electric field. • Plasma behavior in magnetic field. • The effect of the magnetic field on characteristics of the plasma. • Results derived from magnetohydrodynamic equations. • Developments of experimental targets. • Conditions for the existence of a plasma ring. • The theory of stability. • Plasma behavior in traps with magnetic mirrors. • Stability of plasmas configurations. • Shear stabilization. • Other plasma instabilities.

Also for you simplicity and detailed explanations we mention Nicholson, D. R. [70] Roberts, P. H. [73] and Thompson, W. B. [84].

A very important introduction of turbulence using a statistical approach is in Tystovich, V. N [87]:

• Comparison of plasma and liquid turbulence. • General problems of the theory of plasma turbulence. • The balance equation for a turbulent plasma. • Turbulent collisions and resonance broadening. • The spectrum and correlation functions of ion sound turbulence. • The spectrum and correlation functions of Langmuir turbulence. • Electromagnetic properties of a turbulent plasma. • The cosmic ray spectrum.

In [38] the authors Goldstone, R. J. and Rutherford, P. H. realize a comprehensive, understandable and advanced introduction to plasmas physics. They use Hamiltonian chaos:
GENERAL PATHS OF REASONING IN MAGNETOHYDRODYNAMICS.

- Introduction to plasmas. • Single particle motion. • Particle drifts in non-uniform magnetic fields. • Particle drifts in time-dependent fields. • Mappings (Hamiltonian chaos). • Plasmas as fluids. • Fluids equations for a plasma. • Relation between fluid equations and guiding-center drifts. • Single-fluid magnetohydrodynamics. • Magnetohydrodynamics equilibrium. • Collisional processes in plasmas. • Fully and partially ionized plasmas. • Collisions in fully ionized plasmas. • Diffusion in plasmas. • The Fokker-Planck equation for Coulomb collisions. • Collisions of fast ions in a plasma. • Waves in a fluid plasma. • Basic concepts of small-amplitude waves in anisotropic dispersive media. • Waves in an unmagnetized plasma. • High-frequency waves in a magnetized plasma. • Low-frequency waves in a magnetized plasma. • Instabilities in a fluid plasma. • The Rayleigh-Taylor and flute instabilities. • The resistive tearing instability. • Drift waves and instabilities. • Kinetic theory of plasmas. • Kinetics effects on plasma waves: Vlasov’s treatment. • Kinetic effects on plasmas waves: Landau’s treatment. • Velocity-space instabilities and nonlinear theory. • The drift-kinetic equation and kinetic drift waves.

A key step is contained in Stix, T. H. [80]:

I. Wave normal surfaces: Introduction. • The susceptibility and dielectric tensors. • The dispersion relation. • Polarization and phase relations. • Cutoff and resonance. • Wave normal surfaces. • Transition of shapes and labels.

II. Waves in a cold uniform plasma: Introduction • Clemmow-Mullaly-Allys diagram for single ion species plasma. • Propagation parallel and perpendicular to B0. • Hydromagnetic waves of Alfven and Astrom. • Ion cyclotron waves. • Ion cyclotron waves. • The hybrid resonances. The Altar-Appleton-Hartree dispersion relation. • Parallel current flow.

III. Causality, acoustic waves and simper drift waves: Introduction. • Nonlocal behavior. • The electrostatic and electromagnetic approximations. • Finite parallel electron temperature. • Thermal corrections and ion acoustic waves. • Lower-hybrid waves. • Drift waves.

IV. Energy flow and accessibility: • Introduction. • Energy transfer. • Energy density, energy flux and group velocity. • Energy transfer for electrostatic waves. • Some geometrical waves. • Surfaces of constant phase and Lighthill’s theorem. • Ray tracing in inhomogeneous media. • The kinetic equation for waves. • Amplitude transport for a well-defined wave packet. • Radiation transfer. • Accessibility. • Calculation of accessibility.

V. Kruskal-Schwarzschild solutions for a bounded plasma: Introduction. • The boundary equations. • An equilibrium solution. • Linearization of the equations. • Solution of the first-order boundary equations. • Solution of the first-order plasma equations. • The Rayleigh-Taylor instability.

VI. Oscillations in bounded plasmas: Introduction. • Alfven and ion cyclotron waves in a cylindrical plasma. • The vacuum and the boundary equations. • Solution of the steady state problem. • Forced oscillations. • Toroidal eigenmodes. • Density of the modes. • Resonance cones.

VII. Plasma models with discrete structure: Introduction. • The two stream instability. • The beam equations. • The Dawson modes for a plasma of many beams. • The trapping of charged particles. • A nonlinear plasma wave. • Beam-exited plasma oscillations.

VIII. Longitudinal oscillations in a plasma of continuous structure: Introduction. • A physical picture of Landau damping. • Landau damping as viscosity. • The plasma kinetic equations. • A simple kinetic model of Landau damping. • Environments for valid Landau damping. • The collisionless Boltzmann equation. • Analytic continuation of the integrals. • The dispersion relation. • The Van Kampen modes. • The Nyquist criterion for
instability. • The two stream instability in a hot plasma. • Electrostatic waves in a Maxwellian unmagnetized plasma. • The plasma dispersion plasma. • Asymptotic behavior of the dispersion function.

IX. Absolute and convective instability: Introduction. • An intuitive picture. • Further analysis and discussion. • Pulse shape, convective and absolute instability. • Convective instability and amplifying waves. • Absolute instability by residue theorem.

X. Susceptibilities for a hot plasma in a magnetic field: Introduction. • A physical picture of cyclotron damping. • Electromagnetic trapped particle modes. • Solution of Vlasov equation. • Transformation from Lagrangian to Eulerian coordinates.

XI. Waves in magnetized uniform media. Introduction. • Introduction. • Propagation parallel to Bo. • Cyclotron harmonic damping. • Transit time damping. • Propagation perpendicular to Bo, $\omega \neq n\Omega$. • Propagation approximately perpendicular to Bo, $\omega \approx n\Omega$. • The marginal state for the magnetosonic wave. • Power absorption by collisionless processes. • Cyclotron overstability due to pressure anisotropy. • Electrostatic waves in a magnetic field.

XII. Effect on waves from weak collisions: Introduction. • Random walk in velocity space. • Model Fokker-Planck equation; decay of singular perturbations. • The function $C(\xi)$. • Gyrophase and gyrocenter diffusion. • Conservation of momentum. • Damping of Alfven waves. • Particle conservation; electrostatic waves. • Hybrid Resonances. • Stabilization of simple drift waves.

XIII. Reflection, absorption and mode conversion. Introduction. • Zeros and infinities in the refractive index. • Solutions to the wave equation near a turning point. • Asymptotic solutions. • The Budden tunneling factors. • The absorption layer. • Applicability of singular turning point theory. • Mode conversion: the Alfven resonance. • The hybrid resonance. • The standard equation. • The ICRF equation. • The low frequency Alfven resonance. • Matched asymptotic expansions.

XIV. Non-uniform plasmas: Introduction. • The Vlasov equation. • The electrostatic approximation. • Susceptibilities. • The drift kinetic regime. • Small Larmor radius kinetic theory of drift waves. • Drift wave instability. • Flute like drift waves.

XV. The straight trajectory approximation: Introduction. • A long wavelength loss cone instability. • The straight line trajectory approximation. • The enhanced straight line trajectory dispersion relation. • Ion Bernstein waves. • Short wavelength loss cone instabilities. • Drift cyclotron instability. • Drift cyclotron loss cone instability.

XVI. Quasilinear diffusion: Introduction. • Quasilinear analysis. • Conservation of energy and momentum. • Quasilinear evolution. • Cross-B transport. • Wave associated drag. • Collisional relaxation and rf current drive. • Stochasticity. • Superadiabaticity. • Anomalous viscosity for parallel current.

XVII. Quasilinear diffusion in a magnetized plasma: Introduction. • Cyclotron heating. • Heating in tokamak geometry. • Rf induced radial transport in tokamaks. • Quasilinear diffusion in a magnetic field. • Wave associated drag. • Electromagnetic quasilinear theory. • Resonant particle diffusion. • Test particle Fokker-Planck equation. • The Coulomb diffusion coefficients. • Steady state solution for $f(v)$. • $f(v)$ for steady state isotropic ion injection. • Superadiabaticity and decorrelation.

XVIII. Bounce average quasilinear diffusion: Introduction. • Bounce averaging. • Particle conservation in E, $\mu$ coordinates. • The bounce average integrals. • The phase integral.
Further material [11, 19, 22, 25, 30, 34, 35, 37, 42, 50, 53, 65, 75, 77, 96]

The source of lineage of paths of reasoning in MHD: Lectures notes of schools and reviews.

Perhaps one of the main streams in our subject was given in The Culham Summer School on Plasma Physics. Gill R. D. (editor) [36] offer a carefully chosen set of lectures notes from the period 1978-1980:

Section I. Introduction: • Nuclear fusion. • Introduction to plasma physics.

Section II. Theory: Magnetohydrodynamics. • Particle orbit theory. • Plasma waves. • Kinetic theory. • MHD stability theory. • Plasma radiation.

Section III. Advanced theory: Microinstabilities. • Plasma turbulence. • Anomalous transport theory. • Nonlinear laser plasma interaction.

Section IV. Experimental devices: Tokamak confinement devices. • Stellerator confinement devices. • The next generation tokamaks. • Fusion reactor studies.

Section V. Heating and diagnosis: Neutral injection plasma heating. • The theory of radio frequency plasma heating. • Radio frequency plasma heating experiments. • Plasma diagnosis using lasers. • X-ray and particle diagnosis.

In The Nagoya Lecture Notes in Plasma Physics and Controlled Fusion Ichikawa, Y. H. and Kamimura, T. editors [48], we encounter the subjects:

• Development of nuclear fusion research. • A few perspectives in nonlinear dynamics. • Time dependent drift Hamiltonian. • Adventures in magnetohydrodynamics. • Introduction to the theory of fluid and magnetofluid turbulence. • Topics in magnetic reconnection and transport in fusion devices. • Turbulent transport in tokamaks. • Advanced fusion reactors.

Also we mention:

Advanced School on Waves and Instabilities in Plasmas September 1993, International Centre for Mechanical Sciences in Udine, editor Cap, F. [21].


THE FRONTIERS OF RESEARCH.

1. Mathematical paths of reasoning in MHD.

The use of functional analysis is extensive in Lifschitz, A. E [62], where the writer study mainly spectral problems arisen in magnetohydrodynamics, it is demonstrated that the ideal MHD equations can be obtained by means of a variational principle from the actional functional whose explicit form is found, the use of advanced mathematical concepts like Poincare’s map, manifolds with magnetic properties is wide, a carefully description of waves also is given.

An extensive use of Fourier, Laplace and Hilbert transforms is in Jones, W. D. et al. [54]. Tensor calculus is reviewed in Hazeltine, R. D. and Meiss, J. D. [44]. In Abdullaev, [1] we have methods to study Hamiltonian systems and mathematical aspects of Dynamical Chaos. The use of intrinsic coordinates is a key ingredient in Mercier, L and Luc. L. [63].

Heisenberg-Weyl groups, the theory of operator symbols, Lie groups, Lie algebras, representations and the Weyl symbols calculus appears in Tracy, E. R. et al [86]. In Yoshizawa, A. Itoh, S-I and Itoh, K. [95], perhaps the main mathematical tools are: Kolmogorov’s spectrum, renormalization and statistical methods, bifurcation and systems with hysteresis, self-organized dynamics, statistical picture of bifurcation and transition probabilities.

2. Tokamaks and confined plasmas: [1, 6, 10, 17, 29, 45, 63, 68, 69, 79, 82, 83, 90, 91, 92, 93, 97].

3. Technology of nuclear fusion: [24, 31, 32, 39, 41, 78]

4. Turning the wheel of turbulence and chaos in MHD: [15, 28, 33, 43, 46, 55, 59, 61, 67, 94, 95]

5. Waves in MHD: [2, 3, 4, 5, 9, 52, 54, 72, 81, 86].

6. Anomalous transport in MHD: [8].

7. Computational MHD: [10, 12, 17, 40, 51, 58].

8. Further material: [14, 27, 16, 27, 30, 47, 71]

9. Conclusion.

Finally we are in the frontiers of research in nuclear fusion, just go to: https://fusion.gat.com/theory/Home.

Bibliography.

84. Thompson, W. B. An Introduction to Plasma Physics. Pergamon Press 1964